Listening for algebraic expressions in the hands of deaf learners

Lulu Healy, Solange Hassan Ahmad Ali Fernandes, Fabiane Guimarães Vieira Marcondes and Heliel Ferreira dos Santos

Bandeirante University of São Paulo, Brazil

We offer this contribution to the theme: Student mathematics learning and experiences in multilingual classrooms. The paper aims to explore the mathematical practices of deaf learners. More specifically, we seek to identify how learners whose first language is LIBRAS (Brazilian Sign Language), a visual-gestual language, express mathematical generalisations. We begin by considering briefly how language has been conceptualised in deaf education historically, before introducing constructs from Vygostsky's work which have informed our research activities. Finally, we present snapshots of the practices which emerged as a group of deaf learners worked with activities involving generalisation in pattern activities, and consider the evidence in their mathematical expressions of algebraic thinking.

LANGUAGE AND DEAF LEARNERS

We live in a plural world, a world comprised of a diversity of cultural groups and a multitude of languages and linguistic practices. Indeed, language is a central aspect in defining cultures, as every language, be it verbal, gestural, pictorial or of any another form, is representative of a particular social group. Language is also central to teaching and learning, and to the development of school cultures, themselves composed of participants from multiple language backgrounds. In mathematics classrooms, alongside the natural languages, learners also have to contend with the discourse of mathematics itself. The complexities associated with attending to different linguistic practices at times result in tensions within the school context, and this is certainly in the case when we consider the experiences of deaf learners.

In the historical trajectory associated with the education of deaf learners, language, or more precisely the type of language to be used in educational practice, occupies centre stage. The debate is frequently polarised in terms of use of manual (signed) languages verses oral methods, a polarisation which can be traced back at least to the 18th Century. On one side was the Frenchman, Abbé de L'Epée, who, based on the signs used by the deaf people of Paris, elaborated a formalised sign system, *Methodical Sign*. In 1760, he founded the first public school for the deaf in France, the "Institution Nationale des sourds-muets" in which this system was used. On the other side was the German educator Samuel Heinicke, who, at the same time, was developing an oral/aural method to teach deaf people to speak. He was strongly opposed to the methods used by L'Epée and the ideological split was born, as evidenced in the correspondences between the two (Moura, 2000). After L'Epée's death in 1798, defenders of oral methods continued to criticise approaches which privileged the used of sign languages. And, almost a century later, in 1880, at the Congress of Milan, as Lang (2003) describes, participants "voted to proclaim that the German oral method should be the official method used in the schools of many nations" (p.15). He also reports that deaf people were excluded from this vote. Oralism hence became the dominant method and, for many years, deaf learners were discouraged or even forbidden from using sign language during schooling.

It was only in the 1960s and 1970s that this dominance began to be challenged, particularly after the scientific recognition of American Sign Language (and consequently the sign languages of other countries throughout the world) as a true and natural language (Stokoe, 1960/2005)[1]. As Oralism was questioned, a number of other methods were developed. There is insufficient space to describe all these in this paper, suffice to say that in Brazilian schools today, be they schools for the deaf or mainstream schools including deaf learners, a variety of methods are employed. In the school in which this study was conducted, LIBRAS (Brazilian sign language) is considered as the deaf learner's first language (L1) and written Portuguese as a second language (L2). In mathematics lessons in this school, both languages are simultaneously present, with instruction from the teacher in LIBRAS (sometimes via an interpreter) and written work in Portuguese. We have found few studies in which the use of this Bilingual approach has been investigated specifically in relation to mathematics learning–much more attention has been given to literacy and to the complexity of applying the principles of linguistic interdependence derived from studies of second (spoken) language learning to deaf learners. In this respect, it has been argued that to understand deaf literacy learners, it is necessary to take into account the set of sensory modalities available to them, to ensure they have the opportunity to appropriate and manipulate all possible meditational means at their disposal (Mayer and Akamatsu, 2003). In this paper, we consider the challenge of learning mathematics in LIBRAS. In particular, we ask, how do deaf mathematics learners express mathematical generalities in LIBRAS and whether these expressions evidence modes of algebraic thinking?

MEDIATION, SIGN LANGUAGE AND MATHEMATICS LEARNING

Our approach has been strongly influenced by the debate on the mediating role of material and semiotic tools, especially as we learnt that the Vygotskian construct of mediation has its roots in his work with differently-abled individuals (Vygotsky, 1997). For Vygotsky, the use of language as an instrument in thinking, is central in the ways that learners appropriate–*make their own*–the forms of acting which characterise the social groups to which they belong. Vygotsky worked with deaf

[1] Today it is generally accepted that the manual modalidade is as good a medium for language as the oral modality (see, for example, Goldin-Meadow, 2003).

students during the reign of Oralism. Characteristically, his perspective on the education of learners with special needs was before its time and, despite the centrality of language in his ideas, he was not a defender of the Oralist tradition. Rather he argued: “[I]nstruction by this method contradicts the deaf child’s nature. It is even necessary to break the child’s nature in order to teach him speech. Here is the truly the tragic problem of special education for the deaf” (1997; p.124). Instead, Vygotsky saw mimicry[2] as a “genuine language with all the wealth of its functional meaning” (1997 p. 231).

Vygotsky believed that deaf learners should study in mainstream schools, since the only difference between deaf and hearing students is an organ of perception which might be substituted by another instrument. And, although he believed that an adequate substitution would enable deaf students to attain as highly as their hearing counterparts, his position was that it may also mean that they follow different paths to do so, since, just as the inclusion of any other tool in the process of activity alters its entire structure and flow, so too the substitution of the ear by another instrument can be expected to be associated with a profound restructuration of the intellect. Here, we should make very clear that we are not referring to a state of deficiency, but to one of difference. In this vein, to better understand the deaf mathematics learner, we need to better understand what it means to practice mathematics in the medium of sign language, how sign expression link to the other languages present in classroom and how those whose cognitive processes are mediated primarily by a visual-gestual language and only secondarily by a sequential language come to think mathematically.

In the rest of this paper, we present snapshots from a research project in which we examined the interaction of a group of deaf learners with activities involving generalisation. In each example, we seek evidence in the expressions of the students of algebraic thinking. Initially, we borrowed from Radford (2010), a characterisation of algebraic thinking composed of three inter-related elements: A sense of indeterminacy, a form of acting analytically with indeterminate objects and the use of a semiotic system appropriate to support the previous two characteristics. Like him, we accept that the process of expressing generalisation is made up of layers, not organised hierarchically but with its scope “interwoven with the material form that we use to reason and to express the general (e.g. the standard alphanumeric algebraic semiotic system, natural language or something else)” (p.42).

INTRODUCING THE STUDY

Using methods associated with design experiments (Cobb et al., 2003), research activities were developed with the aim of enabling the learners to explore pattern sequences. The snapshots we present in this paper come from the first two of five one and a half hour sessions realised with a class of six deaf students in the ninth year of a

---

[2] According to the translators note, the word “mimicry” is a translation for Russian sign language.

school in the municipal of Barueri, São Paulo[3]. The students were all adults (aged between 18 and 31 years) who studied at night. The group of researchers included the class teacher (Heliel) and in addition an interpreter was present. All the sessions were video-recorded.

Activity 1

Since these students had had no previous experience of working with generalisation patterns, the first activity was planned to verify how they might describe the regularities in a repeating sequence (Figure 1) and if they were able to predict which kind of face would be associated with any position in the sequence.

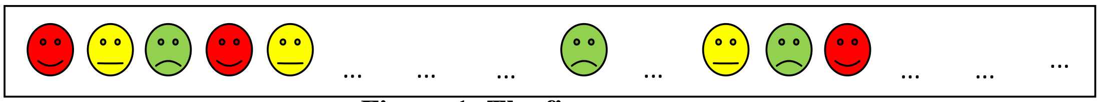
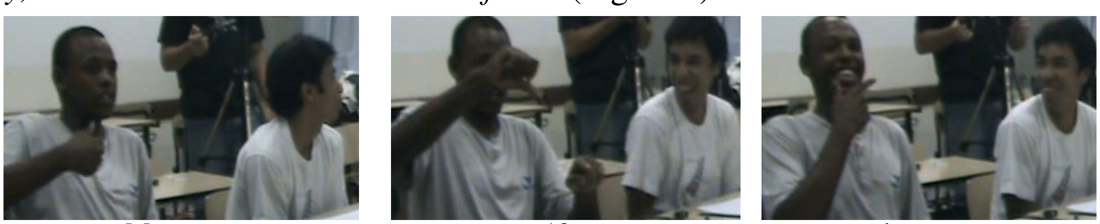


Figure 1: The first sequence

Although the students had some difficulties in understanding questions such as "Which face occupies position 6? And position 11?" in their written form, once they were translated into LIBRAS, they were able to complete the sequence shown in Figure 1. The next task focused on the positions of the sad faces. This proved to be difficult and, although some students developed arithmetic strategies involving adding groups of 3 to offer alternative positions beyond those represented on paper, no-one associated the numbers generated with the multiples of 3. What the students did seem to realise was the existence of a relationship of dependence between the position number and the kind of face – most notably expressed by Felipe[4] who towards the end of the session animatedly signed what we translate as "*I am 19. I am happy, because 18 is sad and comes before*" (Figure 2).



My age | 19 | happy

Figure 2: Felipe's observation

Felipe had developed a method which, in fact, could be used to determine the face of any position. It involved summing groups of three to a known location of a sad face (or sometimes groups of fifteen when the difference was big) and then counting on one for a happy face and two for an indifferent one. So was he thinking algebraically? According to Radford's definition, this depends on whether we consider that he was making use of some sense of indeterminacy, and while his method is general, it always involved working with the determined: He only ever kicked it into action after

[3] This experiment is one of a series being conducted with support from CAPES (Project no. 23038.019444/2009-33).

[4] The names of all the students cited in this paper have been changed.

a specific position was given. Yet surely Felipe's strong personal sense of the dependent relationship between dependent and independent variables is an important aspect here and while his method would probably never engender itself to expression in conventional algebra, it would be possible to express the indeterminate object, perhaps in a kind of algorithmic form, that started with a mantra of "given any number". This activity, however, had not motivated such a move. We also note that the structure of LIBRAS is perhaps further removed from the highly sequential and compressed structure of algebraic language than is natural spoken language.

As we reflected on the student's strategies during this activity, we decided that for the second activity we would make the aim of constructing a general rule more forefront. We also decided that instead of a repeating pattern we would use a sequence in which the structure of this rule was more made more explicit, in both the narrative of the task and in the numerical sequence associated with this narrative. Taking into account an observation by one of the students at the end of the first session – who suggested perhaps that Lulu was the mother of both Heliel and Fabiane – Activity 2 was elaborated.

Activity 2

Activity 2 presented Heliel and Fabiane as the children of Lulu. On Day 1, Heliel had three strands of hair. Each day, another three hairs grew. Fabiane had five strands of hair, which also increased at a rate of a further three per day. The new task consisted of producing a way of calculating the number of strands of hair for each child using the day as the independent variable (see, Figure 3). As the students began to complete the first table by adding 3 the number of hairs the previous day, Breno perceived that the number of hairs could be calculated by multiplying the number of days by 3. As he explained this to the rest of the group, he never actually mentioned the number of days, choosing instead the example of the 4th day, the generality was communicated in natural language through the use of the sign for "always" (Figure 4).

HELIEL							
DAY	1	2	3	4	5	6	7
STRANDS OF HAIR	3	6	9				

FABIANE							
DAY	1	2	3	4	5	6	7
STRANDS OF HAIR	5	8	11				

Figure 3: The second sequence

3x4=12 Always... ... times 3.

Figure 4: Breno's method

This method was readily appropriated by the other students, who together completed the table in Figure 5. Still the independent variable was not explicitly mentioned, with suggestions that the rule was applied to *any* day–with various symbols offered by the researchers to represent this term–met by indications from the students of the particular day they would use.

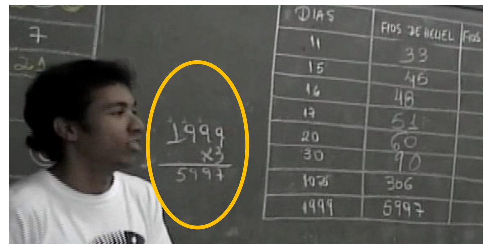


Figure 5: Completing the table

As the students moved onto to consider Fabiane's hairs, again the first strategy was to consecutively add threes. This is not really surprising, the tabular presentation and the narrative of growing hairs, both likely to favour such a strategy. Indeed, it is well known that the recurrence strategy is also common among hearing students (Stacey, 1989). Following a variety of attempts by the researchers to draw students' attention to the functional relationships inherent in the sequences–using dramatisations, rhythmic actions, deitic gestures and the like–Breno noticed that, on each day, Fabiane had two more hairs than Heliel (Figure 6).

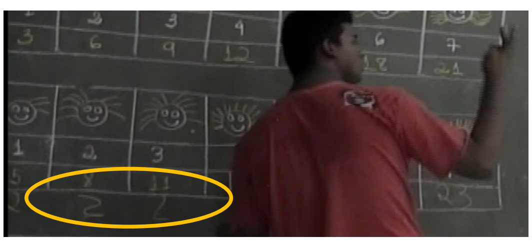


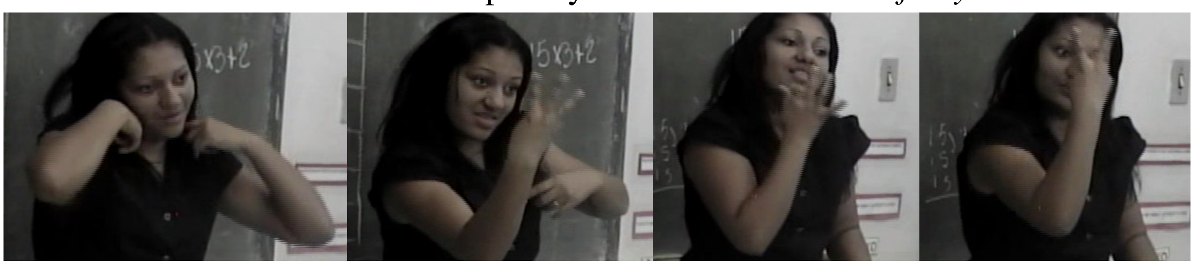
Figure 6: Always add 2

The next step was to use this new way of looking at the two sequences in order to complete the table shown in Figure 5. Instead of just adding two to the values previously calculated for Heliel, this provoked a new round of discussion about a possible rule for calculating Fabiane's head of hair. Various attempts were made by different students, and again the researchers' interventions were directed at encouraging the students to focus on the relationship between independent and dependent variables. It has to be said that the process of intervening was by no means straightforward. It is not easy to induct hearing students into operating with unknown objects, it is harder still when interventions have to be translated into LIBRAS by an interpreter who is not a mathematics teacher. Nonetheless, communication was possible and eventually Ana asked if she could explain her thinking to everyone, offering to complete the second line of the table which involved calculating the number of hairs for 15 days. She first wrote an arithmetic expression, 15x3+2, on the board. However, when she explained this expression, she added the word "always" before each operation. We believe this was her first attempt to communicate the generality of her method (Figure 7).

15 always times 3 always add 2

Figure 7: Ana's explanation

As Ana went on to the 16th day, she added another layer to her explanations. This time she signed "*Heliel always times three, Fabiane always add two*". We would argue that the inclusion of the names of the "babies" (she used the signs of the real people in her expression), is another indication of the generality that she has perceived (Figure 8). Frustratingly for us perhaps, although quite clearly not for Ana, she still felt no need to mention explicitly the variable *number of days.*



Heliel always times 3 Fabiane always add 2

Figure 8: A general rule

REFLECTIONS ON THE LEARNERS' DEVELOPING GENERALISATIONS

Thus far we have presented the mathematical generalisations of a group of deaf learners as they interacted primarily using LIBRAS. We note that, as the two sessions progressed, there was an increasing sense of generality in the methods that they developed and in their propensity to look for ways of relating two sets of variables. However, in terms of the sense of indeterminacy cited by Radford (2010) as a critical element in algebraic thinking, our evidence suggests that neither of the two tasks presented in this article, nor the interventions of the researchers, were sufficient to encourage the inclusion into the discourse of the learners practices which made visible indeterminate objects. Perhaps one of the problems that the students experienced was that the narrative of the task was expressed in natural language (LIBRAS) and through numerical values, but not visually. If the new hairs had been visually grouped in sets of three, perhaps the structure of the general term would have been more evident. Yet despite the absence of a visual structure and although the students found the activities challenging, in both, they did develop methods which enabled them to calculate any case we gave them. It was just that the methods they expressed did not include mention of the unknown. Perhaps it could be argued that the simultaneous form of information transfer favoured by LIBRAS is even more distant from that sequential form of algebra than is spoken language, but actually Ana's expression quite closely resembles a spoken language generalisation. What we would argue is that the problem was not only that the students were not familiar with a semiotic system in which indeterminate numerical objects can be expressed readily, it was also that, strictly speaking, they came to view the goal of the activity as a challenge to come up with methods of calculating a set of values (essentially what they were asked to do!), and hence it was perhaps not so surprising that they were not motivated to include an aspect of a new mathematical discourse into their practices. Radford (2010) argues that we cannot separate the scope of a mathematical generalisation from the tools through which it is expressed. We would agree, but, would add that, whether students interact in spoken or signed language, the challenge of finding contexts in which the students interpretation of the activities goal motivate the appropriation of such tools still remains. This we see as our next step.

REFERENCES

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Goldin-Meadow, S. (2003). *The resilience of language: What gesture creation in deaf children can tell us about how all children learn kanguage*, Psychology Press, a subsidiary of Taylor & Francis, New York.

Lang, H.G. (2003). Perspectives on the history of deaf education. In M. Marschark & P. E. Spencer (Eds.), *Oxford Handbook of deaf studies, language, and education* (pp. 9 –20). New York: Oxford University Press.

Mayer, C. & Akamatsu, C.T. Bilingualism and literacy. In In M. Marschark & P. E. Spencer (Eds.), *Oxford Handbook of deaf studies, language, and education* (pp. 136-147). New York: Oxford University Press.

Moura, M. C. (2000). *O Surdo, caminhos para uma nova identidade.* Rio de Janeiro: Editora Revinter.

Radford, L. (2010). Layers of generality and types of generalization in pattern activities. *PNA – Pensamiento Númerico Avanzado*, 4(2), 37-62.

Stacey, K. (1989). Finding and using patterns in linear generalising problems. *Educational Studies in Mathematics*, 20, pp. 147-164.

Stokoe, W. C. (1960/2005). *Sign language structure: An outline of the visual communication system of the American deaf.* Studies in Linguistics, Occasional Papers 8. Buffalo, NY: Department of Anthropology and Linguistics, University of Buffalo. Reprinted in *Journal of Deaf Studies and Deaf Education, 10,* 3–37.

Vygotsky, L. (1997). *Obras escogidas V – Fundamentos da defectología*. Traducción: Julio Guillermo Blank. Madrid: Visor.