MATHEMATICAL OPPORTUNITIES FOR STUDENTS WITH DISABILITIES

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As in many other countries throughout the world, the trend to include learners with special educational needs within the mainstream education system began in earnest in Brazil during the latter part of the 1990s. As attention to the rights of this group of learners has increased, so too has their participation in the Brazilian educational system. In 1998, there were a total of 337,326 students with special educational needs enrolled in Brazilian schools. According to data from the 2010 school census, by 2010, this number had more than doubled to 702,605. Even more strikingly, whereas, in 1998, only 48,923 of these students were included in mainstream classes in regular schools, in 2010, this had grown almost 10-fold to 484,332.

In the light of this context, how to ensure that these learners have opportunities to a mathematics education with respects the particular ways in which they experience and participate in the world has become a present and pressing question. Our research[1], began as a response to the pragmatics demands of mathematics teachers who wanted to better understand how to include blinds students, deaf students and students with physical and/or cognitive disabilities in their classrooms. These teachers expressed many concerns. Working with students with disabilities was not something that had been addressed in either initial or in-service education courses, leaving them feeling ill-prepared and uninformed. They reported having little, or no, access to specifically adapted pedagogical resources and pointed to difficulties in locating textbooks in Braille or materials directed at students who use the Brazilian sign language, LIBRAS, Another difficulty that the teachers experienced was in finding literature from which they might learn of successful strategies of inclusion in mathematics classrooms in Brazil. On top of all this, there were also deep concerns about what inclusion actually means in the context of mathematics education. One teacher put his problem thus "*When I was faced for the first time with a blind student in my classroom, I thought I am not a good enough teacher to deal with this situation. I already have problems with students who see. How can I teach anything to someone who cannot.*"

As our work has progressed, so too has our contact with teachers of mathematics within specialized as well as mainstream schools. They also express similar worries about the lack of materials and research specifically addressing learning processes of

[1] Throughout the paper, I will draw on research carried out by myself and other members of the research programme *Rumo à Educação Matemática Inclusive*, PROESP-CAPES). In particular, I would like to acknowledge the contributions of Solange Hassan Ahmad Ali Fernandes in all aspects the empirical work described in this paper.

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the students with whom they work. Irrespective of the kind of schools in which mathematics is being studied, it seems that the development of a more inclusive school mathematics depends on deepening our understandings of how mathematics practices and knowledge are mediated through different sensory channels. Perhaps by doing so, we might not only become better able to create learning opportunities for students with disabilities, we might also build more robust appreciations of the relationships between experience and mathematical cognition more generally. The rest of this paper outlines our attempts to contribute to these objectives.

AVOIDING DEFICIT MODELS OF DIFFERENCE

One of our first tasks was to seek a theoretical basis for our research. We were drawn to the socio-cultural perspective of Vygotsky and his colleagues for a number of inter-related reasons. First, in his work in the area of what at the time was called *Defectology*, Vygotsky (1997) warned against focusing on quantitative differences in achievements between those with and without particular physical or cognitive disabilities. Instead, he proposed a more qualitative approach which involves considering how and when the substitution of one tool by another may empower different mediational forms and hence engender different practices (Healy & Fernandes, 2011; Healy & Powell, in press). His approach stressed the potential for development of learners with disabilities, rather than positioning them as deficit in relation to some supposed "norm". In relation to empowering those without access to one or other sensory field to participate in social (cultural) activities, for Vygotsky the solution lies in seeking ways to substitute the traditional means of interacting with information and knowledge with another. For example, he suggested that the eye and speech are "instruments" to see and to think respectively, and that other instruments might be sought to substitute the function of sensory organs (Vygotsky, 1997). Vygotsky's writings suggest that he was attributing, at least implicitly, to organs of the body—more specifically, to the eye, to the ear and to the skin—the role of what he latter denominated psychological tools. And in this sense, as well as empowering their users to participate in otherwise inaccessible activities, substitute tools they can also be expected to restructure the activity in question:

> "...by being included in the process of behavior, the psychological tool alters the entire flow and structure of the mental functions. It does this by determining the structure of a new instrumental act, just as a technical tool alters the process of natural adaptation by determining the forms of labor operation". (Vygotsky 1981, p.137).

Vygostsky's stance suggests that to understand the mathematics learning of those with disabilities, we need to understand how the particular set of material, semiotic and sensory tools through which they attribute meaning to their activities motivate different forms of participation in mathematics. On its own, though, this will give us only one part of the picture. As the above quote stresses, tools do not only shape the meanings that become associated with particular activities, they also shape the activities themselves: the relationships between tools, activity and thinking are reciprocal. This would suggest that as well as recognising that there are different ways by which mathematical knowledge might be appropriated that depend, for example, on whether or not we have access to the visual feel, or on whether we speak with our mouths or with our hands, it is also necessary to recognise the mathematical knowledge itself might undergo some transformation. Moreover, accepting sensory apparatuses mediate mathematical activity adds force to the argument that cognition is embodied, that the way we think cannot be separated from the way we act and that both have their bases in our body, its physical capacities and its location in space and time (Barsalou, 2008; Gallese and Lakoff, 2005). Hence our theoretical framework combines socio-cultural concerns which initiated in the first half of the last century with more contemporary approaches from embodied. To illustrate very briefly how this developing framework is informing our search for a more inclusive school mathematics, the rest of the paper focuses upon the mathematical practices of one blind student as he used his hands rather than his eyes to see.

FEELING MATHEMATICS

In contrast to vision which is synthetic and global, touch permits a gradual analysis, from parts to the whole (Ochaita & Rosa, 1995). Our work is suggesting that this makes the activity of seeing with one's hands a different cognitive practice from seeing with one's eyes and that the properties of the 'seen' object that are privileged may not necessarily be the same in both cases. Lucas's investigations of activities related to the transformation reflection provide a case in point (more details can be found in Fernandes & Healy, 2007). Lucas became completely blind at the age of two. He had already completed High School within the mainstream school system when he participated in the research and he was familiar with a variety of geometrical objects and relations, although he told us he had never studied symmetry or geometrical transformations.

Figure 1: Lucas constructs an axis

He worked on a series of activities involving the transformation reflection during three sessions, each of about an hour and a half. This example is drawn from the second session, during which he was asked to construct the axis of reflection of a number of pairs of symmetrical segments (an example is shown in Figure 1).

After a number of such tasks, Lucas indicated that he had invented a general method by which he might construct an axis of reflection of any figures with axial symmetry. The researcher asked him to explain to her this method, in a way that she might enact the method on an imaginary geoboard displaying two symmetrical segments.

Lucas: Take as your base, one of the extremities of each of the segments.

Res: Any one?

Lucas: The two extremities on the same…sides

Lucas seemed to be aware that this description was not very precise and, as he explained which points he was referring to, he placed his hands as if they too were symmetrical around an imaginary vertical line in the middle of the board. He then traced the imagined symmetrical segments, stressing, with an extra pointing gesture at the end of the movement, the two the extremities, symmetrical in relation to the imaginary vertical axis as shown in Figures 2 and 3. His gestures suggest that as he mentally reenacts his previous activity, he re-evoked the same cognitive resources as in the initial the concrete doings. He then continued onto the next step.

Lucas: You centralize the axis of symmetry on the midpoint between one extremity and the other of each segment.

Res: And how do I find the midpoint?

Lucas: You could use a ruler, but it's simpler to count the pins between one of the segments and the other and localize the pin that will be the midpoint with the same distance between one segment and the other. Then fix the elastic and trace out a line always obeying the distance, for the two extremities, but also so that the other points on the segments keep the same distance from the axis.

Figure 2: Lucas places his hands ready to trace two segments

Figure 3: Lucas taps his two fingers to indicate symmetrical end points

A striking feature of Lucas's method is its dynamic nature. This dynamism is evident in the way he represents the imagined symmetrical segments as he tries to respond to the researcher's first question and in how he treats the axis of reflection as a dynamic trajectory, constrained so that the distance between it and any two symmetrical points belonging to the segments always have the same distance from the line being traced. Dynamic gestures in which blind students re-enact previous tactile explorations as they abstract mathematics relationships appear seem to be rather characteristic of their interactions with geometrical objects (other examples are available in Healy & Fernandes, 2011). Perhaps it is because of the way Lucas had moved his hands over the materials that he talks of line segments as both collections of points and as trajectories – a view which contrasts those usually expressed by sighted students, who tend, at least initially, to treat segments as whole objects (Laborde & Grenier, 1988; Healy, 2002). Moreover, whereas sighted students tend first to focus on the properties within particular objects, Lucas, in common with other blind students we have worked with, begins by looking for the relative positions of the geometrical points that constitute the objects in a mathematically structurable space. Piaget and Garcia (1989) define the first perspective as *intrafigural* and the second as *interfigural*, arguing that they represent the first two of three hierarchically organised epistemological phases through which mathematical ideas develop. That is, their view is that all learners necessarily pass through a stage of intrafigural analyses before reaching the interfigural stage. Our work with blind students suggests, however, that the perspective that comes to be adopted depends on the available means for mediating the ideas and that it would be a mistake to expect those who do not see with their eyes to necessarily follow the same learning trajectories as those who do.

There is one more point to be made before finishing this brief account. As we watched Lucas's explorations, we noticed that symmetrical hand movements were extremely frequent. Again, this form of exploring figures turned out not to be limited to Lucas, but characteristic of the other blind students who participated in the study. This was not something that had been anticipated in the design of the tasks—since they had been developed on the basis of research into sighted learners' understandings of symmetry and reflection (Kuchemann, 1988; Grenier & Laborde, 1988; Healy, 2002). This stresses how a tendency to design learning scenarios for the blind relying exclusively on what we know about the learning trajectories of sighted might not offer the best opportunities for mathematics learning. Moreover, by concentrating more specifically on the how they use their hands to conceive mathematical objects, we are beginning to recognize how very intimate the relationship between bodily groundings and mathematical abstractions is.

ENCULTURATION AND EMPOWERMENT

To end, it seems appropriate to return to our socio-cultural beginning. According to this perspective, mathematics learning can be defined as appropriating the artefacts and practices that historically and culturally represent the body of knowledge associated with mathematics. A danger with this definition is that it might be taken to imply that learning involves an exclusively one way-process of enculturation into the dominant culture (Gutiérrez, 2010). At its most extreme, this might imply that learning to succeed means learning to be like those idealized in the dominant culture. For the learners with whom we work, this would involve a denial of their very identity. Rather than ensuring opportunities for mathematics learning, if enculturation becomes imposition, then those already marginalised can be expected to be ever-increasingly so. The search for a more inclusive mathematics education hence requires that appropriation is not viewed as a one-way process. Rather, it can be seen as a kind of entanglement of perspectives on an activity, out of which emerge new forms of thinking about the objects in question for, at least, some of those involved (Healy & Powell, in press). The social and the individual are both fully present in this entanglement: the activities undertaken and the expressions associated with them being essentially social acts, mediated by *all* the means available to those interacting within the setting in question – not only the material resources and semiotic presentations, but also the bodily resources and ways of being associated with the multiple identities which the learners bring to the setting. Ensuring opportunities hence involves respecting and encouraging diversity in mathematical practices and avoiding the assumption that everyone will, or should, appropriate mathematics in the same way.

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