RELATIONSHIPS BETWEEN SENSORY ACTIVITY, CULTURAL ARTEFACTS AND MATHEMATICAL COGNITION

Lulu Healy
Universidade Bandeirante de São Paulo

Solange Hassan Ahmad Ali Fernandes
Colégio Nossa Senhora do Rosário

To explore the construction of mathematical meanings in learning settings, we focus on the co-ordinations of speech, gestures, material objects and sensory activities in a dialogue between a mathematics teacher (researcher) and a blind student. We argue that as the student came to know the mathematical object in question (a pyramid), the teacher too engaged in a process of re-conceiving the object through the hands (eyes) of the student. We argue that in the search for a more inclusive mathematics education, we need to pay more attention to ways that students who lack access to one or other sensory field, both to create more accessible learning situations and to extend our understanding of the interplays between perception and conception.

INTRODUCTION AND BACKGROUND

It can hardly be controversial to claim that we develop and that we learn by interacting within the various biological, social and cultural systems that make up the world as we experience it. Individuals construct their own meanings of the mathematics they encounter which depend upon the ways and means through which they come in to contact with the knowledge culturally labelled as mathematics, as well as upon their individual resources – physical, visual, auditory and cognitive. And yet the precise nature of the relations between perception, cognition and culture has long been an academic battleground. We might go back to Plato who distinguished between that perceived through the senses and that perceived by the soul, judging the first untrustworthy and deceptive and the second the intellectual route to the universal forms representing the true essence of reality. A perspective countered by Aristotle, who argued that our knowledge initiates from that which comes to us first in the form of sensations from our sense organs. As we fast forward to the Enlightenment, the debate continues, with the Empiricists, defending the idea that all knowledge is a consequence of experience and the Rationalists insisting that the rules of reason are the means through which knowledge comes to the mind and soul. Forward again to the 1960s and 70s and the emergence of Mathematics Education as an academic discipline, it was Piaget's constructivist perspective, which emphasises the learner as a rational being whose activities are guided by her (mental) logical structures, which dominated research efforts. Finally, as we move into current times, the focus has moved from the individual learner to learners interacting in cultural settings. While in the past, the controversy concentrated on the relationships between experience and intellect of the individual, today cultural and semiotic considerations vie for centre stage, and yet as embodied cognition too enters onto the

2009. In Tzekaki, M., Kaldrimidou, M. & Sakonidis, C. (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. XXX-YYY. Thessaloniki, Greece: PME.

scene, the relationship between sensory activity and knowledge remains a relatively open problem.

It is perhaps surprising given this background that so little attention within the mathematics education literature has focussed on the learning of those who lack access to one or other sensory field. In part, our work with blind mathematics learners is a response to this absence and motivated by our belief that if we are able to identify the differences and similarities in the mathematical practices of those whose knowledge of the world is mediated through different channels, then we may build a more robust understanding of the relationships between experience and cognition more generally. But this is not the only motivation. Like many other countries across the world, current Brazilian education policies legislate for "inclusive schools", aiming towards a system in which learners with particular educational needs are no longer educated in special schools, but attend regular classes. In Brazil, what this means in practice is that mathematics teachers face the challenge of teaching in classes in which, amongst the around 40 students, there might be a small number of students who are blind or deaf or have some other physical or cognitive impairment. Unlike some other countries, in Brazil, most blind students attending the regular public school system will not be accompanied in their mathematics classes by individual support workers nor will the mathematics teacher have extra classroom assistance (although they should have access to a special needs teacher responsible for translating material to and from Braille). In most cases, the mathematics teacher will also have little or no access to materials adapted for blind learners and, almost certainly, have received no training for working with students with special needs (for more details, see, Fernandes and Healy, 2007). Given this scenario, there is also a pressing pragmatic motivation for our research: if we are to create truly inclusive mathematics classrooms, it is critical that we seek to better understand the particularities of learning mathematics of all our students.

MEDIATION AND MULTIMODALITY IN MATHEMATICS LEARNING

In seeking theoretical grounding for our work, we have been influenced by the debate on the mediating role of material and semiotic tools and by approaches which recognise the situated and embodied nature of cognition. In terms of the first, which posits a reciprocal relationship between tools and thinking, we note that the Vygotskian construct of mediation (Vygotsky, 1997) has its roots in his work with differently-abled individuals. He argued that "the eye, like the ear, is an instrument that can be substituted by another" (p.83), and that, just as the inclusion of any other tool in the process of activity alters its entire structure and flow, so too the substitution of the eye by another instrument will cause a profound restructuration of the intellect. Indeed, we already know that there are some differences in the ways that blind learners process data when compared with sighted students (Ochaita & Rosa, 1995). When exploring a physical object or a raised representation on paper, it is the hands of the blind learner which act as substitutes for the eyes. Like the eyes of the sighted, the hands are moved in an intentional manner, catching particularities of the form in order to perceive – and at the same time conceive – the object, although in a slower and successive form. Whereas vision is synthetic and global, touch permits a gradual analysis, from parts to the whole. One question, then, to be addressed in this paper is how this form of exploration mediates mathematics learning, highlighting (or not) particular mathematical relationships and properties or particular ways of thinking about mathematical objects.

To some extent, the second area of influence emerges as a consequence of the first: if we accept that tools are part and parcel of cognitive activities and that the organs of the body can be thought of as mediating tools, then it follows that thinking cannot be separated from the activity in which it occurs nor can it be considered an entirely cerebral activity. In this sense, our approach concurs with that of Radford (in press), who argues that thinking "cannot be reduced to that of impalpable ideas; it is instead made up of speech, gestures, and our actual actions with cultural artifacts (signs, objects, etc.)… thinking does not occur solely *in* the head but also *in* and *through* a sophisticated semiotic coordination of speech, body, gestures, symbols and tools". Indeed, Healy and Fernandes (2008) reported how a blind student coordinated his interactions with his teacher, other blind students, unit cubes and physical representations of geometrical figures to create a sign (in the form of a repeatedly used gesture) to represent his thinking about area and volume, in a multimodal form which linked his perceptual activities with the cultural conceptions of these mathematical objects. Radford (2002) terms this process of coordination, which involves becoming increasingly aware of a cultural object, *objectification*, a construct somewhat reminiscent of Mason's (1989) view of abstraction as a "delicate shift of attention" (p.2) in which learners move between seeing and using expressions as processes or objects.

The aim of instructional situations, within this perspective, becomes the objectification of cultural knowledge, with the means being the signs and materials already impregnated as it were (at least for the teacher) with a cultural history of the objects in question. For Radford, the role of the teacher becomes that of mediating a kind of alignment between the subjective meanings attributed to the objects by the learners and their cultural meanings, by involving their students in an active re-interpretation of the signs in play. From our point of view, however, the role of the (sighted) teacher of blind students is a little more complex: they must also be open themselves to re-align their own perspectives on the mathematical objects according to the alternative perceptions of learners who process data in a manner different to that they are accustomed to. The second question we examine in this paper, then, is how the process of coordinating the physical and semiotic resources within such an instructional setting contributed to the construction of meaning for a three dimensional geometrical object, the pyramid. We concentrate particularly on the central role of one blind student's gestures in this process.

THE RESEARCH CONTEXT

This research was carried out as part of a research project investigating the processes by which blind learners appropriate mathematical knowledge[1]. The project took place in a school from the public school system of the state of São Paulo in Brazil, with a long history of including learners with visual impairments, and counted on the collaboration of group of six teachers from the school, along with a total of twelve blind or partially sighted students who participated in the various empirical activities. Here, we report on an activity in which a student we shall call André (an 18 year-old, second year high school student) attempted to construct his own representation of a pyramid with a square base. It is important to stress that before participating in the project, the blind students had had very little contact with geometrical content in general and André indicated that he had had no prior experience in constructing his own representations of geometrical figures. Before working individually on the pyramid task, he had, however, participated in four other research sessions in which he worked with 5 other blind students to construct representations of plane figures (each of 50 minutes) and a session during which he constructed individually a three dimensional model of a cube (50 minutes). The pyramid task lasted for 40 minutes. All the sessions were video recorded. The way in which the tasks were proposed and the organisation of the students aimed to stimulate dialogues between the participants, including researcher(s) who assumed the role of teacher.

For this article, we have selected, transcribed and coded episodes from a research session in which André, working with one of the researchers, made use of the various resources of the learning scenario.To illuminate the role of gestures in the coordination of resources and examine how they became tools for creating and communicating mathematical meanings, we applied the classification of the four gesture types proposed by McNeill (1992). In this classification, *Iconic gestures* (♦) have a direct relation with the semantic discourse, that is, there exists an isomorphism between the gesture and the entity it expresses, although comprehension of the iconic gesture is subordinated to the discourse which accompanies it. *Metaphoric gestures* (●) indicate a pictorial representation of an abstract idea which cannot be represented physically, for example, when we illustrate with hand movements the limit of a function f(x) when x tends to zero. *Deitic gestures* (☛) have the function of indicating objects (real or virtual) people and positions in space. *Beat Gestures* (♪) are short and rapid and accompany the rhythm of the discourse giving special meaning to a word, not due to the object that it represents but to its role in the discourse.

Representing a pyramid in three dimensional space

To construct his pyramid, André had access to a set of wooden matches intended as possible representations of the edges of the figure and plasticine (modelling clay) for

[1] We are grateful for the research grant from FAPESP (No. 2004/15109-9) which supported this project.

its vertices. Also available were a number of wooden geometrical figures, including the solid square-based pyramid. In his first attempt, André selected just three matches of the same size. These he joined using the plasticine to construct the triangular form presented in Figure 1. Possibly, he had misinterpreted the task as that of producing a two dimensional representation of the three dimensional figure – we had previously noted a tendency amongst most of the visually impaired students to offer representations of just one face of three dimensional figures when asked to produce planar representation (one of the reasons we had resolved to create this new task). To help André perceive the difference between the shape he had constructed and the solid pyramid he had been exploring, the researcher gave him a wooden representation of an equilateral triangle, asking whether a third person would be able to tell which of the two figures his model represented.

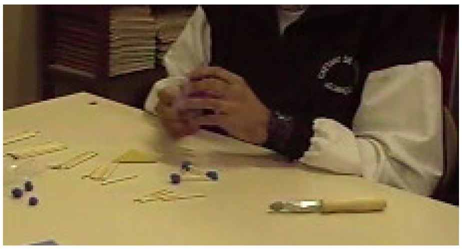

Figure 1 – André's first model

He realised that his representation was closer to the triangle than the pyramid and that he needed to attend to other elements of the original solid:

André: *I thought, like, only of this part here* (☚) (he indicates one of the triangle faces)*, its base is missing, this part underneath* (☚) (moves his hand over the pyramid's square base).

He went on to add a square to one side of the triangle, then moved his hands between the solid pyramid and the new form (Figure 2) for some time. Although not satisfied with his results, he didn't know how to proceed and the researcher intervened.

Figure 2 – Adding the base

André: *I still don't think they are equal*

Researcher: *So what can you do to make it better?*

André: *...I have no idea at all.*

Researcher: *Look, you made this base* (☚) (she puts the square base on to André's right hand), *but the other part* (☚) (places the forefinger of André's left hand on the vertice at the point of the pyramid) *is not in the same plane.*

It seemed that the notion of plane was unfamiliar to André and he continued unsure as to how next to proceed. The researcher attempted to draw his attention once more to the relative positions of the apex of the two forms. Placing André's hand on the plasticine representing the vertex on the triangle not fixed to the square base, she asked him to indicate the equivalent vertex of the wooden solid.

Figure 3 – André's iconic gesture

André did not respond with a deitic gesture, but an iconic one, running two fingers simultaneously along the two edges of one of the triangular faces from the base to the apex, as he affirmed "it's here" (Figure 3).

His gesture indicated that he was seeing the pyramid in a rather different way to that of the researcher. His view was dynamic: the apex was at the end of a process in which André's fingers became successively closer until they met.

More precisely, we now know that what André was in fact feeling was a series of rapidly decreasing squares – for us, a clear indication of how action and thought cannot be separated. Since André was conceiving the pyramid as a collection of layers of decreasing squares, his task was to some how position the top layer (the vertex) without having the layers in between. He made two attempts, first rotating the triangle until it was perpendicular to the pyramid base (Figure 4) in an attempt to construct the pyramid's height, and then, realising the apex was not over the base and unconnected to the other vertices of the base, he pushed the triangle so that it was on top of the square, with both faces once again in the same plane (Figure 5).

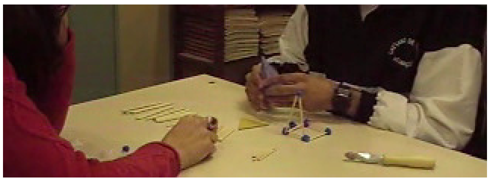

Figure 4 – Giving the pyramid height

Figure 5 – Placing the vertex over the base

Still unsatisfied with the result, he looked to the researcher for help. In the resulting intervention, the researcher supplied a potted history of his activities until that point, still stressing her point of view centred on the faces and edges rather than André's. Yet as she emphasised specifically the elements he had yet to represent, her intervention enabled André to complete the pyramid to his satisfaction (Figure 6).

> **Researcher:** *You found the base and you saw that we have to leave the same plane to make the point* (apex). *We moved out of the plane. Now you have put it in the same plane again. And thinking again, you have made this side* (☛) (moves André's forefinger over a triangle face). *You made this and this* (☛) (drags his finger over the edges of the solid that have corresponding matches in his model). *But there is another here... another here* (☛) (passes André's finger over the not yet represented edges).
>
> **André:** *So I have to put this back up to here* (raises the triangle face once more), *and I have to put a match here and another here* (♦) (he draws in the edge the missing edges, before adding the appropriate matches to complete the figure shown in Figure 6).

With the completed figure in front of him, the researcher asked a last question, attempting to elicit from André a move from action to articulation:

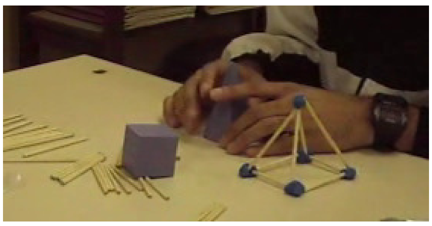

Figure 6 – The final representation

> **Researcher:** *So how would you describe this to colleague, in a letter, who was also working with figures of this form*?
>
> **André:** *I would tell them that the base is square* (☛) *and that as they move up the sides they become smaller until forming a point on top* (♦) (moves his finger simultaneously up the edges of a triangle face).

The researcher was surprised with André's description, in which he finally makes explicit his conception of the pyramid as a pile of decreasing squares. While her previous intervention had successfully drawn André's attention to her way of decomposing the pyramid into faces and edges, it is his previous conception, so difficult for him to model given the available tools, which still characterises his objectification of the geometrical solid. We could say that as the dialogue was played out there was a kind of dual process of objectification – with each of the participants emphasising different aspects of the cultural history of the pyramid, both of which are mathematically valid. The researcher, as teacher, was primarily thinking in terms of the pyramid as it appears in didactical texts for Brazilian school students and where it is important that students come to know facts such as the number of faces and vertices of different geometrical solids. André's conceptions connect to another aspect of the object's history, one that we might trace back to the Ancient Greek mathematician, Eudoxus of Cnidus, who made use of the same idea in order to prove the formula for calculating the volume of a pyramid (Huxley, 1980).

SOME CONCLUDING REMARKS

These two awarenesses of the pyramid emerged, we argue, as the two participants, attempted to coordinate the multimodal forms that emerged during the dialogue. André's gestures hold the key to understanding the particular mathematical properties that became salient to him as his hands were employed as tools for seeing, and his gradual and dynamic exploration of the solid form led him to notice rather different aspects of the pyramid than the synthetic visual feedback received when the eyes are the seeing tools in question. Since it was difficult for him articulate his view with the available modelling tools, it was necessary for him to try to make sense of the researcher's point of view and once again gestures were critical for this activity, since it was the gestures acted out by her that drew his attention to the edges that were missing from his model. It is interesting to note that these edges would in fact have been constructed as a consequence of the decreasing squares' sides had it been possible for André to model more directly how he was seeing the solid.

But it was not only André who had to make changes in his thinking about the pyramid. The researcher too was confronted by a reading of the object that she had not expected as André's description drew her attention to a property of the pyramid that she did not have in mind. Like André, she too became involved in active re-interpretations of the different semiotic signs expressed in a variety of modes, as a kind of reciprocal process of re-aligning personal, other and cultural meanings was played out.

Before we end, it is important to stress how very challenging the task of building a representation of a geometrical solid was, not only for André, but for the other blind students with whom we worked. And although our work is still at an exploratory stage – we have worked with only a small number of students and there are currently very few other studies of blind mathematics from which we can draw – the evidence that we have collected does suggest that André's decomposition of a three dimensional geometrical figures is a common strategy. From the point of view of our pragmatic considerations, we can begin to draw some lessons from our analyses. First, it is clear that as teachers (and researchers) our thinking about the tools we should provide is strongly mediated by our experiences as and with sighted learners. We did not think, for example, that the task might have been more accessible to the blind if it had been suggested that they use modelling clay alone in a first attempt to build their own model of the solid. Or indeed had we considered confection of a multitude of different-sized geometrical figures which might be used as layers. And here we identify a real difficulty in terms of any realisation of a truly inclusive mathematics classroom: unless we develop more robust understandings of alternative (though valid) ways of expressing mathematics, then blind learners, or any other learners who do conform to the supposed educational norm, will be expected to assimilate to existing practices and we will lose the opportunity to exploit the new ways of doing and representing mathematics that they can teach us about.

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