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# SEEINGS AND KNOWINGS OF BLIND STUDENTS: SEPARATE POLES OR TWO SIDES OF THE SAME COIN?

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*The aim of this paper is to analyse how blind learners manage the conflicts between "seeing" and "knowing" in relation to two-dimensional representations of two geometric solids (a cube and a square-based pyramid). It seeks to locate elements within their interactions which make up the repertoires of "knowing" of those who do not see with their eyes, treating the processes involved in such interactions as acts of perception, with their origins in the body, and which serve a mediating role between environment, culture and brain.*

*Keywords*

## INTRODUCTION

Since we began our work with blind mathematics learners, we have been interested in identifying both differences and similarities evident in the mathematical practices of those with and without access to the visual field. One of the points that the results of our research emphasizes is that, in terms of mathematics learning, it should not be assumed that lack of access to one or other perceptual channel will necessarily have a negative impact.

As we began our investigations, we came across a question raised originally in the 17th Century in a letter sent by William Molyneux to John Locke. The question posed by Molyneux, which captured the interest of various philosophers of the time, was whether a person, born blind, who had learned to distinguish between a sphere and a cube through touch, would be able to correctly identify these solids by visual means if he or she gained the capacity to see (Riskin, 2002). A number of quite different responses were offered and amongst the major factors which distinguished between them were the different values attributed to experiences of a sensory nature and, principally, the influence of the body on human cognition. Locke answered negatively in his *Essays on Human Understanding*. His view was that the qualities that come to be associated with certain objects depend on the bodily organism through which they are sensed. Hence, the experiences of a blind individual that result from exploring spheres and cubes through touch would be different from those associated with seeing the shapes through the eyes (LOCKE, 1991). Leibniz in *New Essays on Human Understanding*, on the other hand, answered the question in the affirmative. For him, touch could offer tactile images to blind individuals that may or not coincide with the exact ideas that she or he holds. However, these ideas would be sufficient to enable the individual, on becoming able to see, to discern that a sphere has no distinct vertexes, whereas a cube has eight (Riskin, 2002, p.24).

In 1749, Molyneux's problem came into Diderot's hands. That same year, he published the *Letter on the blind for the use of those who see* which presented his answer. His position was that the blind person who gained the ability to see, would first need to learn to see (just as previously he or she had learned to touch). Moreover, he argued that the ability to distinguish or not the sphere from the cube with time would depend on characteristics of the individual involved. He refered to Saunderson, a blind mathematician and professor of the University of Cambridge who lived between 1682 and 1739, suggesting that if it were he who gained his sight, then, first of he would recognized a square and a circle thanks to the proprieties that he learned through touch. The process of replacing the circle for the sphere and the square by the cube, however, would require time and additional experience.

The points of view of these and others philosophers offered a starting point for this article from which we might interpret the positions of more contemporary theorists in the field of mathematics education.

## THE POLES OF *SEEING* AND *KNOWING*

In our attempts to understand and interpret the activities of blind learners with three-dimensional shapes, we came across the study of Parzysz (1988) and the distinction he made between the poles of *seeing* and *knowing*. Our view is that as we look at this distinction with Molyneux's question in mind, the poles approximate to Leibniz's distinction between the products of the senses, *images* and *exact ideas* that are constituted by properties of objects and by definitions. According to Parzysz (1988), as a learner tries to represent a three dimensional object, he or she confronts a dilemma between representing what is seen and what is known. In this view, representing what is presented to the eyes corresponds to the *seeing* pole, while the *knowing* pole consists in representing the properties of the object that the learner judges to be relevant, that is, in building representation based on cognitive aspects. In relation to the blind learner, there is, at least theoretically, no need to rethink the *knowing* pole – a visual impairment does not imply any kind of cognitive deficiency – but the notion of *seeing* needs to be extended and considered as including also images which result from tactile explorations made with the hand. Here, we are using the term "image" as employed by Damásio (2005), who defines it as a mental pattern in any sensory modality. Actually, although beyond the scope of this article, seeing space also involves the processing of auditory data by the blind individual. Here, however, since we are primarily concerned with seeking to understand the images associated with the tactile explorations of three dimensional shapes, we limit our considerations of the seeing pole to those images associated with touch.

The results of Parzysz's research indicated that in the two-dimensional representations of three-dimensional geometric objects produced by sighted students *knowing* predominates over *seeing* (Parzysz, 1988). More particularly, he identified three attitudes that occur in succession as students move up the school system, but which coexist in certain levels of education. Initially, there is a stage in which, in the representations they produce, there is no evidence that students experience any conflict between *seeing* and *knowing* or if they do, the conflicts are ignored and the students draw what they see. In later stages, the representations show signs that *seeing* is coming increasingly under the influence of *knowing*, as students seek to represent the proprieties of the object that are judged to be important rather than the representing the object exactly as it is sensed.

Our aim in this article is to analyse how blind students manage the poles of "*seeing*" and the "*knowing*" in the representation of two geometrical solids as raised drawings. Given the particularity of the subjects, we seek, in their interactions with the materials and with the researchers, elements that could indicate their repertoires of "*knowing*" and how these elements impinge on the way they see, or whether, for those who are blind, it is the pole of *seeing* which dominates.

## PHENOMENOLOGY AND THE EMBODIED PERSPECTIVE

Thinking in terms of the poles *seeing* and *knowing* seems appropriate in principal in the case of blind learners, but the intimate relationships that exist between the *seeing* pole and the perceptive channels through which visual data is processed led us to attribute a phenomenological character to this notion. That is, to associate the parameters determined by Parzysz with a phenomenological perspective which posits that "it is not we that interfere with things: it is they that show themselves to us, or better, reveal themselves to us" (Carmo, 2000; p.22). According to the phenomenologist Merleau-Ponty (2006), when we come into contact with an object, our perceptive consciousness enables us to perceive the object in total harmony with its form. This quotation invites a reflection about the body and its relationships with the world that surrounds it. Indeed, the relationship between body and cognition deserves particular attention in the case of the blind, if we accept that only that which is tangible by one's body makes up one's perceptive field. In today's terms, this leads us to ideas that are currently characterising the notion of embodied cognition. For Barsalou (2008), for example, an object, once it has been perceived becomes part of our memory. More precisely, for him, this means it comes to compose a repertoire that he describes as a multimodal representation. Having been composed during experience, this memory set becomes available for reactivation in situations of simulation. For Barsalou, simulation is the (re)creation of perceptive, sensory-motor and introspective states acquired during interactions involving the world, the body and the mind.

Here, it is necessary to pause a moment to consider what we mean by perception. For Damásio (2005), perception is not only a question of acts in which the brain receives direct signals from a determined stimulus. He argues that the body should not be seen as passive but as in perpetual interaction with the world, "perception is just as much acting on the environment as receiving signals from it" (p.255). Questioning the brain-body dualism, he presents an alternative model of the human mind, in which, given the intensive interaction which occurs between them, he sees the body and the brain as an inseparable and dynamic organism. The organism that they form interacts profoundly with the physical and social environments it inhabits, with these relationship mediated by the movements of the organism and by sensory apparatuses.

For those who are blind (like those who are sighted), interaction with objects results in the mapping of these objects, a mapping which mobilises the brain-body organism in the construction of a mental pattern (image) for the object. Being tied to the biography of the individual who constructs it, the mental pattern associated with a determined object does not necessarily coincide with the images of the same object held by other individuals. Moreover, in a position close to the phenomenological perspective, Damásio (2005) claims that "we don't know the appearance of things" (p.405), rather the images that we have are based on the changes that they promote in our organism. In the case of blind learners, the images of mathematical objects are intrinsically linked to the form by which they have access to these objects as they interact in the various systems – biological, social and cultural – that compose the world they experience. This was one of the points which led us to consider the work of Parzysz, and to formulate the following question: do objects seen with the hands become associated with images which permit their representation in the same forms as objects seen with the eyes?

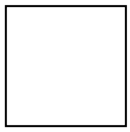
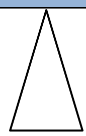
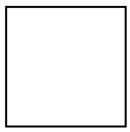
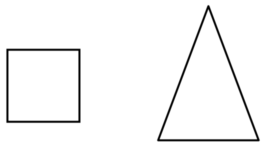
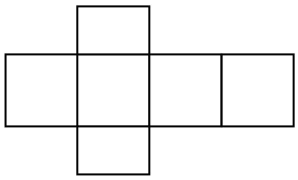
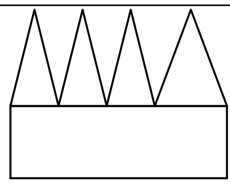
## THE STUDY

The mathematical objects which formed the foci for this study were representations of three-dimensional forms. The study aimed to illuminate our understanding of the development of geometric thinking amongst those without access to visual experiences. Twelve blind and partially sighted high school students from a mainstream school of the public system of the State of São Paulo participated in our study. They were provided with drawing materials and paper adapted for their use. The empirical work occurred in two stages. In the first stage, the learners were asked to draw a square and an isosceles triangle to represent the wooden figures – representation of plane figures, even though in fact they have a three dimensional form – as they explored the shapes with their hands. In the second stage, they were asked to represent on paper two three dimensional shapes, a cube and a square-based pyramid.

The students were divided into three groups, with six students in one group, four students in the second group and two students in the third. Each group worked on the representation activities during three sessions of 50 minutes. The sessions were videoed and all the material produced by the students was collected for analysis. In Parzysz's (1988) study, focus of attention was also on these geometrical objects, although he chose to present to his students skeleton representations (models consisting only of edges and vertices), rather than the solid wooden shapes we used. This choice was mediated by considerations of characteristics of the haptic system and because the results of our previous explorations suggested that the solid forms were more easily identifiable by the blind students than models made up of a collection of elements.

The demands of the task were presented orally to the students and they were asked to draw the geometric forms that they received, first the cube and then the square-based pyramid. They were told that they could produce as many drawings as they like, until they were satisfied that they had produced an appropriate representation. They had access to a range of drawing materials adapted for the blind. We intend to discuss in detail the work of three of the twelve students to illustrate the different strategies that were adopted in their attempts to produce two-dimensional representations of the three-dimensional shapes. Table 1 presented the drawing of these three students (the material used by the students enabled them to produce raised figures, resulting in a tactile drawing, hence Table 1 re-presents their drawing as the raised outlines are not visible in photos).

Table 1: The student's representations

| | CUBE | PYRAMID |
|---|---|---|
| DANI |  |  |
| ANDRÉ |  |  |
| LEANDRO |  |  |

Both André and Dani produced their representation by putting one of the faces of the solid flat on the paper and drawing around it with a drawing tool that produces a raised edge of the outline. In fact, this procedure was used by eleven of the twelve participants. It is worth stressing that since they could not see what the others were doing, the choice of the strategy can be seen as an individual decision, even though all the students in each group were working around the same table.

### Dani's drawings

To draw the square and the triangle in the first stage, Dani positioned one the figures so that one side was parallel to her body and drew round them. When she received the first three-dimensional form, she quickly recognised that it was a cube. Using a similar procedure that had been adopted with the plane figures, she positioned one of the faces of the cube on the paper, so that its edge (and hence another of the faces) was again parallel to her body, before drawing around the shape. After she had finished producing this outline, she explored the impression that she had made on the paper and exclaimed "So a cube looks like this! Like a square!". She decided that she was not satisfied with her design and wanted to redo the drawing (the process of drawing round the shape was not that easy to represent and this first attempt was only square-like). She used exactly the same procedure producing a second squarish shape. This time she was satisfied with the drawing she created.

The next shape she received was the pyramid. Without attempting to name this solid, her first strategy involved placing its square base on the paper. She began the process of tracing around the shape, but stopped after an instant, felt the whole shape again and, as she pointed to the pyramid's vertex, she declared "*But this part won't be there*." For a little while she placed each of the faces (the triangle faces as well as the square base) face down on the paper. As she did this, she recognised the shape as a pyramid. She placed the square base again on the paper, but continued to be worried that the vertex would not figure in her drawing and appealed to the researcher for help:

Dani: This isn't going to be in it. This part here, oh! (places her fingers on the vertex). How will it come out if it is upright here? (points to the vertex again)

Researcher: What I want now is that you draw the figure.

Dani: But this won't be in it. This part here (shows the vertex once more, as in Figure 1a).

Researcher: Won't it?

Dani: Unless I do it like this ... look ... lie it down (she positions a triangle face on the paper as shown in Figure 1b), but, and this here afterwards? (shows the base of the pyramid).

Figure 1a Figure 1b

Figure 1: Dani works with the pyramid

Although it is clear that she is not entirely satisfied, Dani decided to draw round one of the triangle faces of the pyramid.

As we reflect upon her interactions with the shape, we note that she possessed in her memory images (or mental patterns) associated with both the solids, which allowed her to recognise them. In this recognition process, she apparently also sought for particular properties that characterised the shapes for her. Her explorations suggest that in the case of the cube, she recognised the congruency of its faces and the existence of its vertex and base allowed her to name the second solid as a pyramid. That is, in some way, Dani, who was born blind, had access to images of geometrical solids that were evoked on the basis of her knowledge of the tactile properties of the objects in question. This considerations, lead us to suggest that the pole of knowing was present for Dani. That is, she knew about properties that characterised these shapes. In the case of the cube, the fact that she only represented one of the congruent faces did not seem to perturb her – perhaps she felt it served a representative for the others. However, her view of a pyramid, led her to want to present both its base and its vertex in her drawing, but she wasn´t able to place the solid to that they were both on the paper at the same time. That is, her difficulty was to represent the three dimensional object in two-dimensions, and she resorted to seeing only one of the two-dimensional aspects of the shape.

### André's drawings

André treated the square and the triangle in exactly the same form as Dani, positioning the figures so that one of their sides was parallel to his body and drawing around it. Also like her, he was able to recognise both the three dimensional solids he received. He too chose to represent the cube by drawing round just one of its faces (Figure 3) and appeared satisfied with the result.

Figure 2: André's cube

The pyramid, though, was much more difficult for him, evident in his successive attempts to place the pyramid in different ways on the paper. His actions suggested that his conflict centred around the fact that the shape had two kinds of faces, triangle and square, and he wanted both to be included in his figure. In the end, he resolved this dilemma by representing the solid by means of two unconnected plane figures, a representative of the triangle faces and the square base (Table 1). The activities of André indicate that he made use of previously constructed images to identify the two shapes, successfully evoking the pole of *knowing*. Once again, his difficulty was to put in two-dimensions what he was seeing.

### Leandro's drawings

The procedures used by Leandro, along with the drawings he produced, were different to the strategies used by the other students. In the case of the plane figures, he chose not to draw round them, but used his fingers to measure their sides before drawing them using a freehand style. The same drawing was repeated various times on the same piece of paper until he was satisfied that the measures were close enough. His drawings of the three dimensional shapes were particularly surprising.

In the case of the cube, Leandro began by measuring some of its edges with an adapted ruler, counting the number of faces, recognising the form and congruency of each one, before started to produce his freehand drawing. The final drawing is shown in Table 1. It was produced face by face, always maintaining a common edge between two faces. He used his fingers to draw the faces as congruent. In essence, he produced the net of the cube. The strategy that he used to draw the pyramid began in the same manner as the cube. However, having measured the edges and ascertained the number of faces and their form, he began by drawing a relatively large rectangle to represent the square base. He then placed one of the triangle faces flat on the paper, so that its edge coincided with the edge of the rectangle and drew round the triangle face. This process was repeated a total of four times – once for each of the faces (Figure 3).

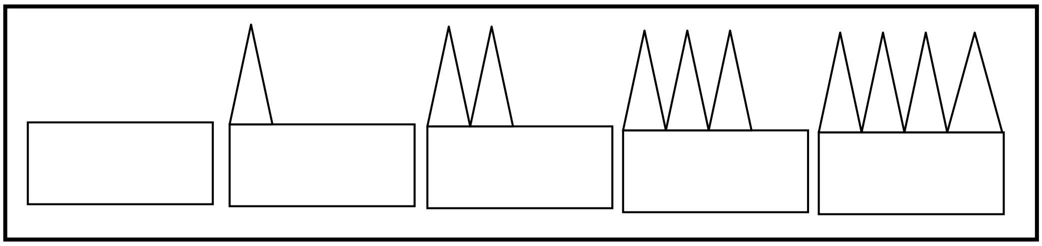


Figure 3: The Leandro's pyramid step by step

Impressed by the resemblance between Leandro's drawing and the nets for these solids, one of the researchers asked if he had previously worked with such nets. Leandro confirmed that he had not; he had simply drawn the shape as he imagined it:

| | |
|---|---|
| Researcher: | Have you already worked with this kind of figure? |
| Leandro: | No. Drawing, I have drawn before, but not this type. |
| Researcher: | But have you worked with nets of solids? |
| Leandro: | No. It was like this that I imagined the shape. |

Like Dani and André, Leandro was also able to easily recognise and name the figures. What distinguished his work from the others, was how he attempted to transfer onto paper his tactile perceptions of the shape. In the case of the pyramid, perhaps there are some indications that the *knowing* pole was influencing his seeing – as he sacrificed the congruency of the edges of the square base, so that he could indicate how all four triangular sides were joined to it. Nonetheless, it is difficult to separate the poles of *seeing* and *knowing* in this case, since this choice was also motivated by his feeling of the common edges.

## SEEING IS KNOWING, KNOWING IS SEEING

The results of this study indicate that the blind students had formulated mental images of particular geometrical figures on the basis of touch. These images were reactivated in order that the figures we gave them to draw could be identified and named. We might then associate these images with the pole of knowing. Certainly, it was the case that the students noticed particular properties that they used for the purposes of classifying. But, images, according to Damásio are mental patterns in some sensory modality – images are also seeing in this view. There is hence a danger that if we separate the students' thinking around the poles of *knowing* and *seeing*, we imply that knowing is somehow stripped of any sensory information. Our results suggest that this is far from true. In relation to the production of two dimensional representations, for the majority of the students who participated in this study their sensory impairment led to the predominance of haptic stimuli as the tasks progressed and, at least at first sight, it seems that their drawings were not always good representations of what they were seeing.

If we return to the idea of phenomenology though, our evidence does suggest that the perceptive conscienceness of the blind students permitted them to perceive and attend to three-dimensional figures as they are: that the shapes revealed themselves in terms of faces edges and vertexes. Where there was a lack of harmony was not between the shapes themselves and the individuals, but, to some extent for both Dani and André, between their bodily resources and the paper-based representations of these objects. They found it difficult to represent in two dimensions what their hands were seeing in three. What they ended up drawing was consistent with what they saw, but fragmented and gave little sense of the whole.

So how might we answer our initial question: do objects seen with the hands become associated with images which permit their representation in the same forms as objects seen with the eyes? We think the answer may be, it depends. As our work with this group continued, we obtained evidence to show that the students had more success when they could build three rather than two dimensional representations (Figure 4). This suggests that the conflict did not revolve so much around *seeing* and *knowing* but around the resources made available to the students and the nature of the mathematical tasks in hand.

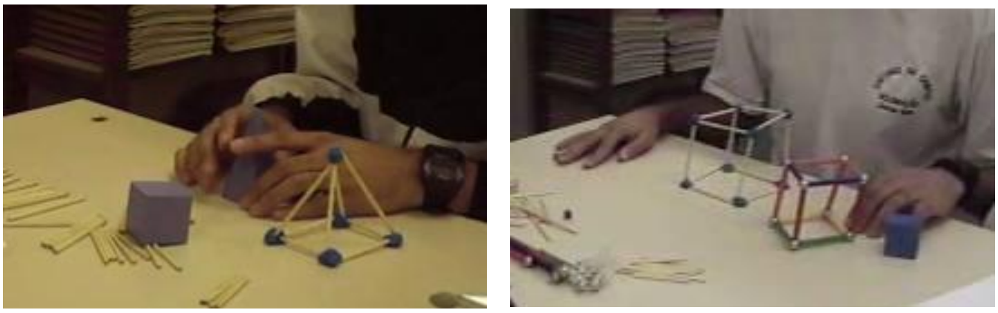


Figure 4: Three-dimensional representations of the cube and the pyramid

Hence, our analysis leads us to suggest that the representations produced by the blind students were influenced by both, knowing and seeing, but that in contrast to the results obtained by Parzysz (1988) in his studies with sighted students, we have little evidence that would allow is to conclude that knowing came to dominate over seeing. In the representations made on paper or in space, the participants in our study, attempted to represent, the objects in the form that they experienced or imagined them – although it was the case that imagining did allow stretching the limits say, in the case of Leandro, as to what might count as a square. This leads us to wonder whether it makes sense to divide thinking into poles of *knowing* and *seeing* at all, or whether it is better to see them simply as two sides of the same coin.

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